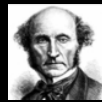


LOGIC

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Topic 5

Venn Diagrams



Summary

In this topic, we will learn ...

- (1) Three kinds of statements that classify things into categories:
 - (a) **Universal** statements
 - (b) **Existential** statements
 - (c) **Singular** statements
- (2) Venn diagrams with one circle.
- (3) Venn diagrams with two circles.
- (4) Venn diagrams with three circles.
- (5) Using Venn diagrams to represent the above three kinds of statements.
- (6) Using Venn diagrams to test validity of arguments.

Universal Statements have the form **ALL A are B**

Example: “**All** lemons are yellow ”

- ◆ Statements of the form “all A are B” are called “**universal** statements” because they talk about **ALL** the things in a category (e.g., all lemons).
- ◆ The various forms of statements below are all logically equivalent to the universal form “All A are B”.

Every A is B

Everything that is A is B

Every lemon is yellow.
Everything that is a lemon is yellow.

No A is not B

Nothing is A but not B

No lemon is not yellow.
Nothing is a lemon but not yellow.

If anything is A, then it is B

If anything is a lemon then it is yellow.

It is **not** true that **some A are not B**

It is not that some lemons are not yellow.

Existential Statements have the form **SOME A are B**

Example: “**Some** lemons are yellow .”

- ◆ Statements of the form “some A are B” are called “**existential** statements” because in modern logic ...

“Some A are B” = “There **exists** at least one A which is B”

“Some lemons are yellow” = “There **exists** at least one lemon which is yellow.”

- ◆ The various statements below are all logically equivalent to “some A are B”.

Something is A and B

Something is a lemon and yellow.

Some B are A

Some yellow things are lemons.

Not all A are not B

Not all lemons are not yellow.

It is **not** true that **all A are not B**

It is not that all lemons are not yellow.

- The *negation* of an *universal* statement (“it is *not* that *all* A are B”) is an *existential* statement (“some A are not B”).
- The *negation* of an *existential* statement (“it is *not* that *some* A are B”) is an *universal* statement (“all A are not B”).

Statements	Universal	Existential	Equivalent			
1. All crows are black.	✓		⊙			
2. All crows are <i>not black</i> (or <i>non-black</i>).	✓			★		
3. Some crows are black.		✓			◇	
4. Some crows are not black.		✓				▲
5. Every crow is black.	✓		⊙			
6. At least one crow is black.		✓			◇	
7. At least one black thing is not a crow.		✓				
8. If anything is a crow then it is not black.	✓			★		
9. Nothing is a crow but not black.	✓		⊙			
10. Some non-black things are crows.		✓				▲
11. Something is a crow and black.		✓			◇	
12. Not all crows are black.		✓				▲
13. Not all crows are non-black.		✓			◇	
14. It is false that some crows are black.	✓			★		
15. It is false that some crows are non-black.	✓		⊙			
16. It is not true that all crows are black.		✓				▲

Singular Statements have the form ***i* is A**

... where the small letter "*i*" refers to a **specified singular object**.

Example: "*Queen Elizabeth II* lives in UK."

- ✦ It is called a "singular statement" because it talks about a specified singular object (e.g., Queen Elizabeth II).
- ✦ Singular statements put specified singular objects into categories (e.g., the above statement puts the Queen into the category of those who live in UK).
- ✦ A singular statement implies the existence of the thing named by the singular term in the statement.
- ✦ In modern logic: "*i* is A" = "*i* **exists** and belongs to the category of A."

Example: "*Cate Blanchett* is Australian but not British."

= "*Cate Blanchett* exists and belongs to the category of Australians as well as the category of non-British."

Summary: Three kinds of statements about categories

Universal Statements

ALL A are B

- = Nothing is A but not B
- = Nothing belongs to ... categories "A" and "not B" at the same time

Existential Statements

SOME A are B

- = Something is A and B
- = Something belongs to ... categories "A" and "B" at the same time

In order to do well at Venn Diagrams

- You must be able to distinguish the three different kinds of statements.
- You must be able to rewrite universal statements into the form "**nothing is ...**"
- You must be able to rewrite existential statement into the form "**something is ...**"

Singular Statements

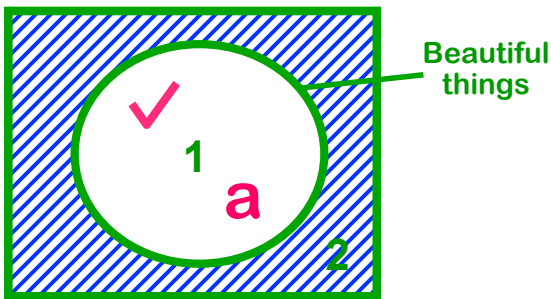
Individual *i* is A

- = *i* belongs to category A

Statements	Universal	Existential	Singular
1. Some dogs like cats.		✓	
2. It is not true that some dogs like cats.	✓		
3. All dogs likes cats.	✓		
4. It is not true that all dogs like cats.		✓	
5. Rex likes cats.			✓
6. It is not true that Rex likes cats.			✓
7. At least one cheap thing is good.		✓	
8. It is false that at least one cheap thing is good.	✓		
9. If anything is cheap then it is good.	✓		
10. It is false that if anything is cheap then it is good.		✓	
11. My lunch is cheap and good.			✓
12. No one believes that my lunch is cheap and good.	✓		
13. Someone believes that all cheap things are good.		✓	
14. Everyone believes that some cheap things are good.	✓		
15. If anyone rebels, then they will be killed.	✓		
16. If someone rebels, then someone will be killed.	✗	✗	✗

Venn Diagrams - with one circle (representing one category)

- ▶ We use a **rectangle** to represent the **world (or universe)**.
- ▶ We use **circles** to represent different **categories**.
Note: we must *label the circles* to show what categories they represent.
- ▶ We **number** all the **regions** in the diagram.
 - Region 1 represents the category of things that are beautiful.
 - Region 2 represents the category of things that are NOT beautiful.
- ▶ We use **shading** to represent **non-existence**.
 That is: *Nothing* exists in a shaded region.
- ▶ We use a tick “ ✓ ” to represent the **existence of something un-specified**.
- ▶ We use a **small letter** to represent the **existence of a specified singular thing**.



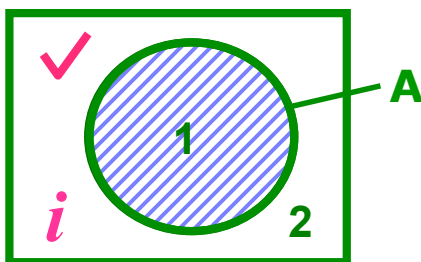
Note: Some books use “*” instead of “✓” to represent existence. In this unit, we use “✓”.

Example: “Everything is beautiful.”
 = “Nothing is not beautiful.”
 = “Nothing exists in Region 2.”

Example: “Something is beautiful.”

Example: “Audrey Hepburn is beautiful.”

- ▶ Use a **rectangle** to represent the **world (or the universe)**.
- ▶ Use **circles** to represent **categories**.
- ▶ **Label** every circle to show what category it represents.
- ▶ **Number** every region in the diagram.
- ▶ Use a **tick** to represent an **existential statement** - indicating the existence of some unspecified things.
- ▶ Use a **small letter** to represent a **singular statement** - indicating the existence of a specified singular individual.
- ▶ Every **universal statement** can be re-written in the form “nothing exists in region(s) such-and-such”.
- ▶ Use **shading** to represent **non-existence**.



Region 1 = A
 Region 2 = non-A

E.g. “Something is non-A”

E.g. “Individual *i* is non-A”

E.g. “All things are non-A”

= “Nothing is A”

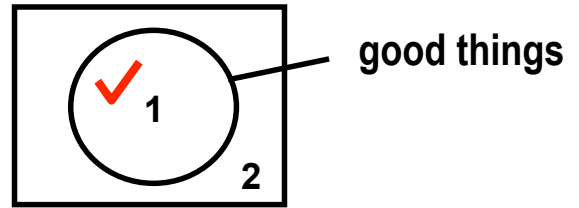
= “Nothing exists in the region(s) representing A”

= “Nothing exists in region 1.”

Represent each statement below by Venn Diagram

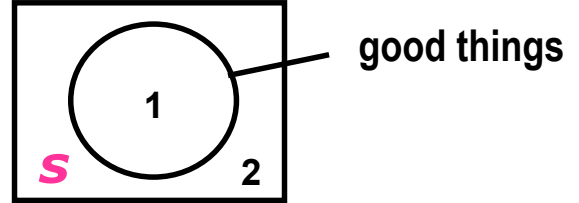
Something is good.

Region 1 = things that are good
Region 2 = things that are not good



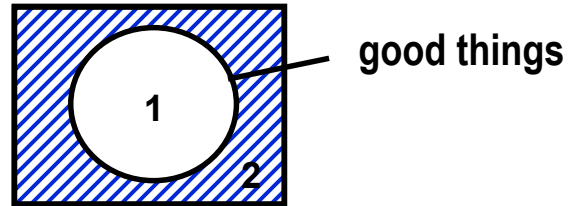
Satan is not good.

Use "s" to represent Satan.



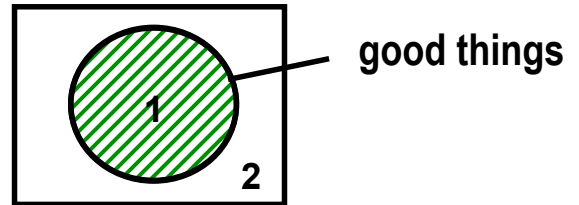
Everything is good.

- = Nothing is not good.
- = Nothing exists in the region(s) representing things that are not good.
- = **Nothing exists in Region 2.**



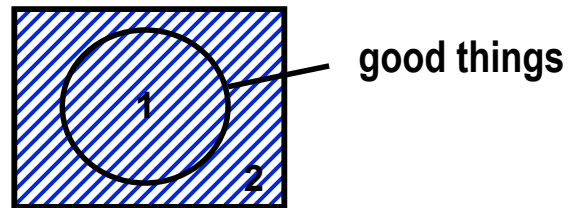
Everything is not good.

- = Nothing is good.
- = **Nothing exists in Region 1.**



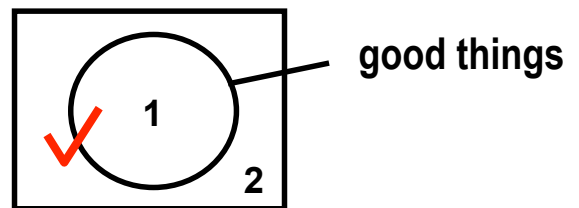
Nothing exists.

- = Nothing exists in any category.
- = **Nothing exists in regions (1, 2).**



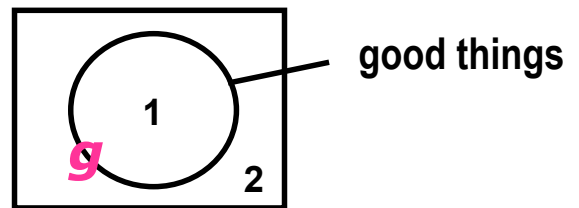
Something exists.

- = Something exists – but it is not known to what categories it belongs.



God exists.

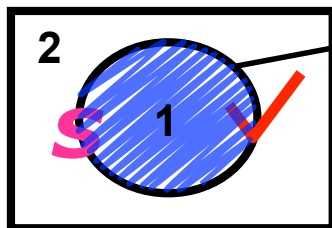
- = God exists – but it is not known to what categories God belongs.



Important Principle #1: When it is **not known** whether some existing thing belongs to (∴ is inside) or does not belong to (∴ is outside) a category, we put the symbol representing the thing **on the border** !

Representing Several Statements in the Same Venn Diagram

- (a) **Something exists.**
- (b) **Satan exists.**
- (c) **Nothing is good.**



good things

To represent (a), put a **tick** on the border between regions 1 and 2.
 To represent (b), put an “**s**” on the border between regions 1 and 2.
 To represent (c), **shade** Region 1.

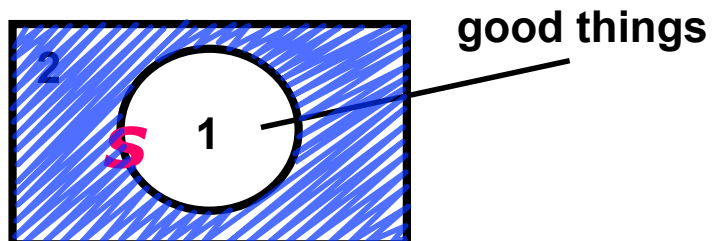
Important Principle #2: **Nothing** can exist in a **shaded** region. So, anything (whether part of a symbol or the whole symbol) that lies inside a shaded region should be treated as **deleted** by the shading. (E.g., If a tick lies half in and half out of a shaded region, then the part lying inside the shaded region should be treated as if it has been deleted. So, the tick **should be interpreted as lying outside** the shaded region.)

- ⊙ The position of the **tick** in the diagram originally means that something exists in either region 1 or region 2. But nothing can exist in region 1 as it has been shaded. So, we must interpret the tick, given the shading, as implying that something exists in region 2 (i.e., outside region 1).
- ⊙ For the same reason, we must interpret the position of the letter “**s**” in the diagram, given the shading, as implying that Satan exists in region 2 (meaning: “Satan is not good”).

Use Venn Diagram to Test Validity

- P1. **Satan exists.**
- P2. **Nothing is not good.**

 C. **Satan is good.**



- ▶ To represent P1, put an “**s**” on the border between Regions 1 and 2.
 - ▶ To represent P2, **shade** Region 2.
 - ▶ Since region 2 is shaded, the “**s**” cannot be located inside region 2. Then, it must be interpreted as locating in region 1 only.
 - ▶ Check whether C is already represented (without any extra drawing)
 - YES! C is already represented – for, as we have seen, “**s**” must be interpreted as locating in region 1 only, which represents “Satan is good”.
- ∴ The argument is **VALID**.

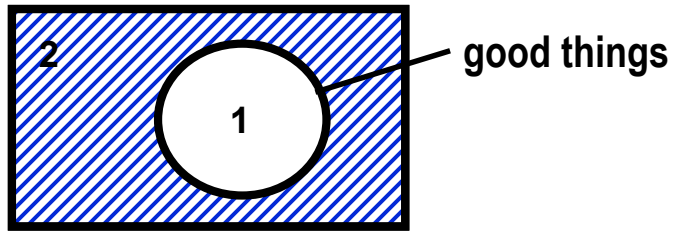
Important Principle 3

An argument is **valid** =_{df} When all the premises are represented in the Venn Diagram (as true), **the conclusion is automatically represented** (as true) too.

Use Venn Diagram to test the validity of the argument below

P. Everything is good.

C. Something is good.



▶ Check whether C is already represented (without any extra drawing)

- For C to be represented, a tick should appear inside Region 1.
- But no tick is actually located there.

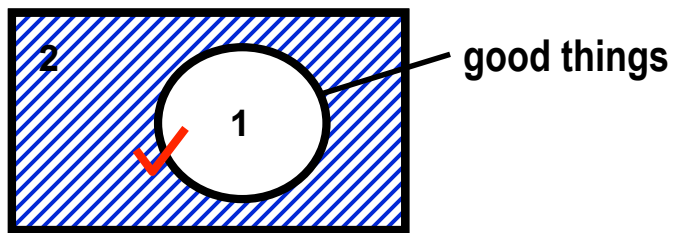
∴ The argument is **INVALID**.

(Note: The above argument is invalid because it has not been assumed that the universe is non-empty. But if we modify the argument by adding to it the assumption of non-empty universe as a second premise (see below), then the modified argument will be valid.)

P1. Everything is good.

P2. Something exists.

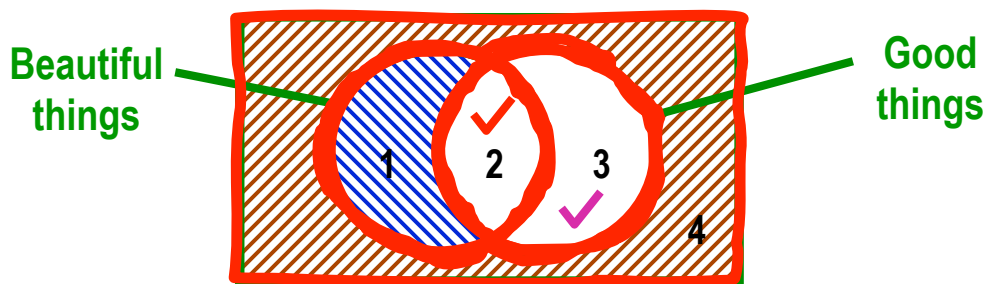
C. Something is good.



C is already represented in the diagram.

∴ The argument is **VALID**.

Venn Diagram - with two circles (representing two categories)



- Regions 1, 2 together represent the category of things that are **Beautiful**.
- Regions 2, 3 together represent the category of things that are **Good**.
- Region 1 represents the category of things that are **Beautiful but NOT Good**.
- Region 2 represents the category of things that are **BOTH Beautiful and Good**.
- Region 3 represents the category of things that are **Good but NOT Beautiful**.
- Region 4 represents the category of things **NEITHER Beautiful NOR Good**.

“Some beautiful things are good.”

= “Something is both beautiful and good.”

“Some good things are not beautiful.”

= “Something is good but not beautiful.”

“All beautiful things are good.”

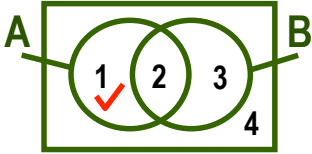
= “Nothing is beautiful but not good.”

“Everything is either good or beautiful.”

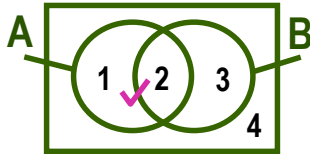
= “Nothing is neither good nor beautiful.”

Class Practice : Represent statements by Venn diagrams

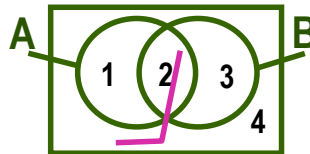
Some A are not B
= Something is A & not B



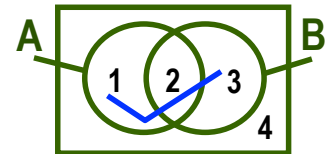
Something is A



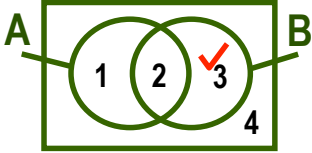
Something is (A & B) or neither
= Something is (A & B) or (not A & not B)



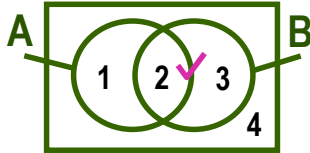
Something is A or B



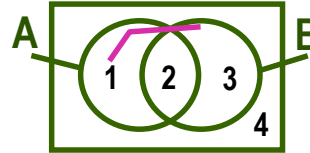
Some B is not A
= Something is B & not A



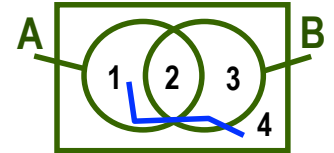
Something is B



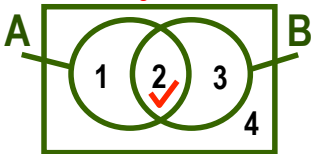
Something is A or B but not both
= Something is A or B & not (A & B)



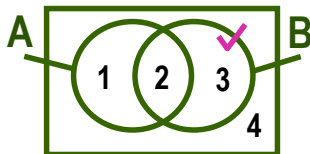
Something is not (A & B)
= Something is not A or not B



Some A is B
= Something is A & B

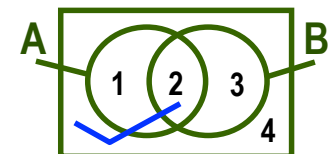


Something is not A

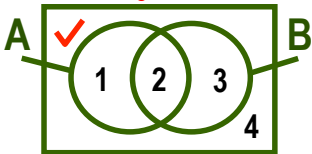


Note: We can use a **crooked line** (instead of a tick) to represent complex cases of existential statements. (A tick, after all, can be understood as a crooked line.)

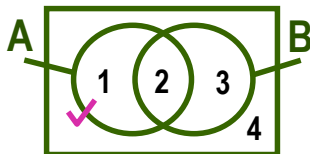
Something is A or not B



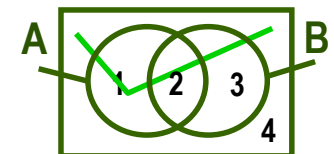
Something is neither A nor B
= Something is not A & not B



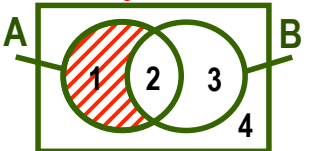
Something is not B



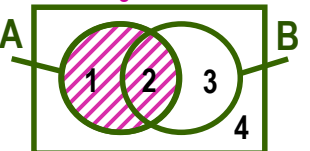
Something exists



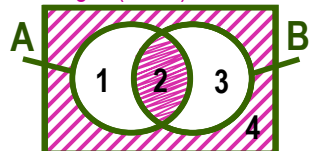
All A are B
= Nothing is A & not B



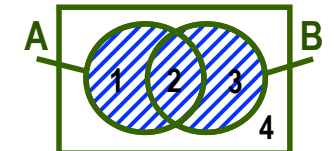
All things are not A
= Nothing is A



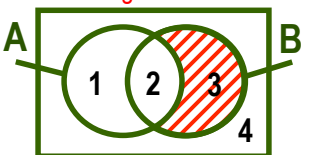
All things are A or B but not both
= Nothing is (A & B) or neither



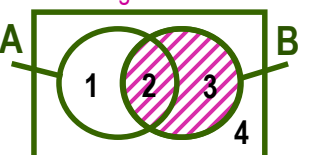
All things are neither A nor B
= Nothing is A or B



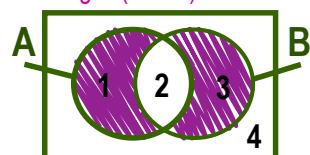
All B are A
= Nothing is B & not A



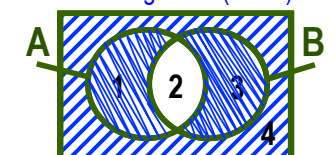
All things are not B
= Nothing is B



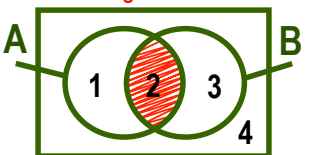
All things are (A & B) or neither
= Nothing is (A or B) but not both



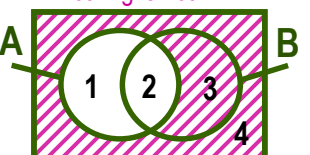
All things are A & B
= Nothing is not (A & B)



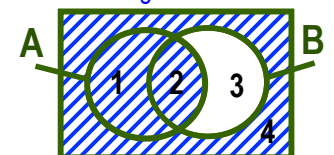
All A are not B
= Nothing is A & B



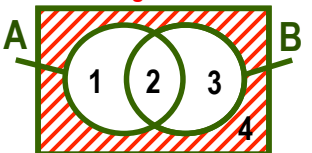
All things are A
= Nothing is not A



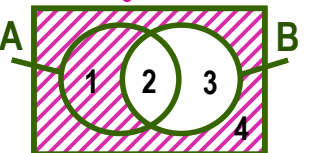
All things are B & not A
= Nothing is A or not B



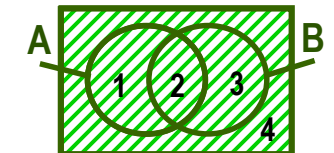
All things are A or B
= Nothing is neither A nor B



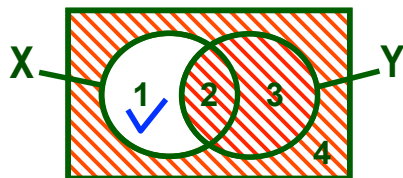
All things are B
= Nothing is not B



Nothing exists



A mechanical procedure to decide what to do to what regions



Basic Categories	represented by	Regions
X		1, 2
Y		2, 3
not X		3, 4
not Y		1, 4

Example 1

Everything is X but not-Y

- = Everything is (X & not Y)
- = Nothing is not (X & not Y)
- = Nothing is (not X or not not Y)
- = Nothing is (not X or Y)
- = Nothing is in ((3, 4) or (2, 3))
- = Nothing is in (2, 3, 4)

► **Shade** regions 2, 3, 4

Example 2

Some X is not-Y

- = Something is (X & not Y)
- = Something is in ((1, 2) and (1, 4))
- = Something is in 1

► **Tick** region 1

Tip #1

Turn **universal** statements into statements of the form “**Nothing is ...**”

Tip #2

“ Everything is A ” = “ Nothing is not A ”

“ All A are B ” = “ Nothing is (A & not B) ”

“ No A is B ” = “ Nothing is (A & B) ”

Tip #3

★ If different sets of regions are connected by “**or**”, then take the **union** of the sets (i.e., **all** the regions involved)

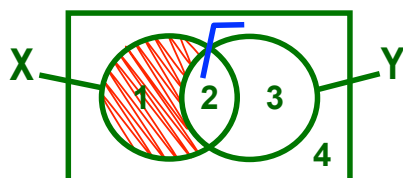
★ If different sets of regions are connected by “**and**”, then take the **intersection** (i.e., the **common** regions)

Tip #4

Turn **existential** statements into statements of the form “**Something is ...**”

Tip #5

“ Some A is B ” = “ Something is (A & B) ”



Basic Categories	represented by	Regions
X		1, 2
Y		2, 3
not X		3, 4
not Y		1, 4

Example 3

No A are not-B

- = Nothing is (X & not Y)
- = Nothing is in ((1, 2) & (1, 4))
- = Nothing is in 1

► **Shade** region 1

Example 4 (more difficult ☹)

Something is either X and Y or neither X nor Y

- = Something is ((X & Y) or (not X & not Y))
- = Something is in (((1, 2) & (2, 3)) or ((3, 4) & (1, 4)))
- = Something is in (2 or 4)
- = Something is in (2, 4)

► **Tick** across regions 2, 4

* Use **one** tick only.

* The tick mustn't enter other regions.

Tip #1

Turn **universal** statements into statements of the form “**Nothing is ...**”

Tip #2

“ Everything is A ” = “ Nothing is not A ”

“ All A are B ” = “ Nothing is (A & not B) ”

“ No A is B ” = “ Nothing is (A & B) ”

Tip #3

★ If different set of regions are connected by “**or**”, then take the **union** of the sets (i.e., **all** the regions involved)

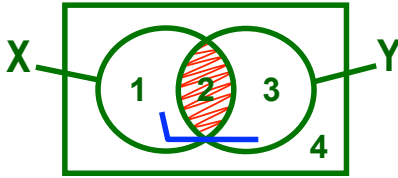
★ If different sets of regions are connected by “**and**”, then take the **intersection** (i.e., the **common** regions)

Tip #4

Turn **existential** statements into statements of the form “**Something is ...**”

Tip #5

“ Some A is B ” = “ Something is (A & B) ”



Basic Categories represented by Regions

X	1, 2
Y	2, 3
not X	3, 4
not Y	1, 4

Example 5

All X are not Y

- = Nothing is (X & not not Y)
- = Nothing is (X & Y)
- = Nothing is in ((1, 2) & (2, 3))
- = Nothing is in 2

► **Shade** region 2

Tip #1

Turn **universal** statements into statements of the form "Nothing is ..."

Tip #2

"Everything is A" = "Nothing is not A"
 "All A are B" = "Nothing is (A & not B)"
 "No A is B" = "Nothing is (A & B)"

Example 6 (more difficult ☹)

Something is X or Y but not both

- = Something is (X or Y) & not (X & Y)
- = Something is (X or Y) & (not X or not Y)
- = Something is in ((1, 2) or (2, 3)) & ((3, 4) or (1, 4))
- = Something is in ((1, 2, 3) & (1, 3, 4))
- = Something is in (1, 3)

► **Tick** across regions 1, 3

- * Use one tick only.
- * The tick mustn't enter other regions

Tip #3

★ If different set of regions are connected by "or", then take the **union** of the sets (i.e., **all** the regions)
 ★ If different sets of regions are connected by "&", then take the **intersection** (i.e., the **common** regions)

Tip #4

Turn **existential** statements into statements of the form "Something is ..."

Tip #5

"Some A is B" = "Something is (A & B)"

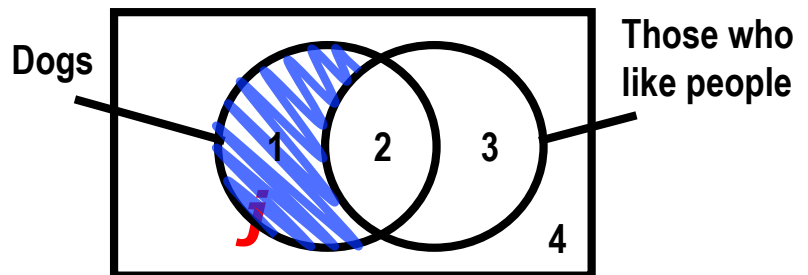
Using Venn Diagram to Test Validity

Example 7

P1. All dogs like people.

P2. Jeff does not like people.

C. Jeff is not a dog.



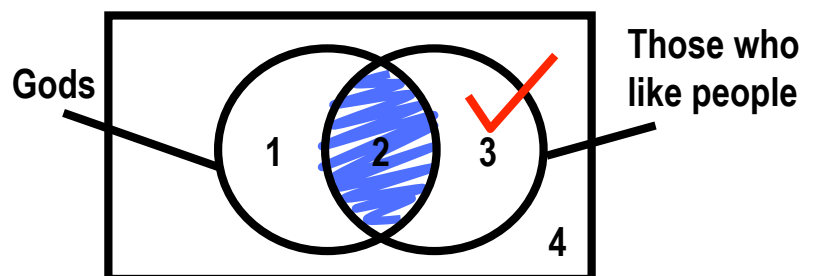
In order for C to be true, "j" should appear **completely outside** the circle representing dogs. That is indeed the case. ∴ The argument is **VALID**.

Example 8

P1. No gods like people.

P2. Something is not a god.

C. Something likes people.



In order for C to be true, a tick should appear **completely inside** the circle representing those who like people.

That is not the case. Therefore, the argument is **INVALID**.

Use Venn Diagram to test whether the argument is valid

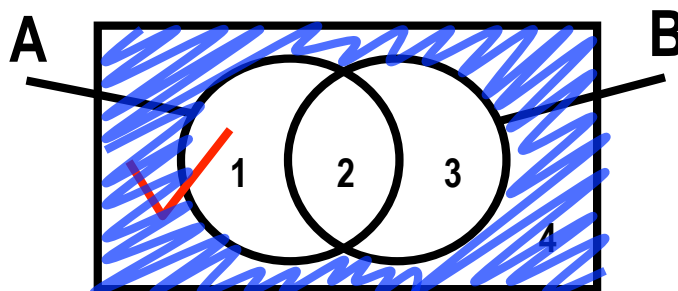
Example 9

P1. Something is not B.

P2. Everything is A or B.

C. Something is A.

VALID



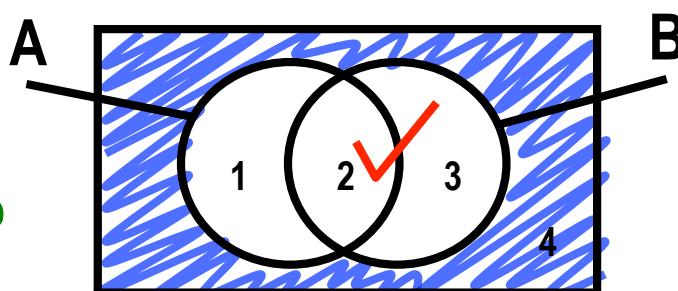
Example 10

P1. Something is B.

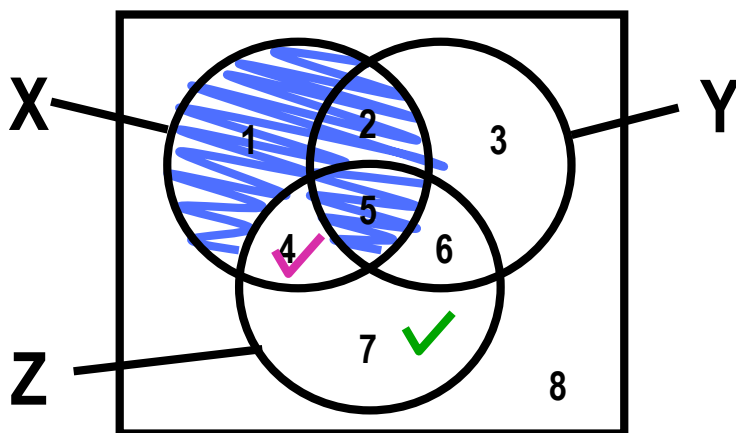
P2. Everything is A or B.

C. Something is not A.

INVALID



Venn Diagrams - with three circles !!!



(i) **Some Z are neither X nor Y**

(ii) **Some X are Z but not Y**

= Something is ($X \& (Z \& \text{not } Y)$)

= Something is in Region 4.

(iii) **All X are Z and not Y**

= Nothing is ($X \& \text{not } (Z \& \text{not } Y)$)

= Nothing is ($X \& (\text{not } Z \text{ or } Y)$)

= Nothing is in Regions 1, 2, 5.

Categories

Categories	Regions
X	1, 2, 4, 5
Y	2, 3, 5, 6
Z	4, 5, 6, 7
not X	3, 6, 7, 8
not Y	1, 4, 7, 8
not Z	1, 2, 3, 8

$Z \& \text{not } Y$

4, 7

$X \& (Z \& \text{not } Y)$

4

not Z or Y

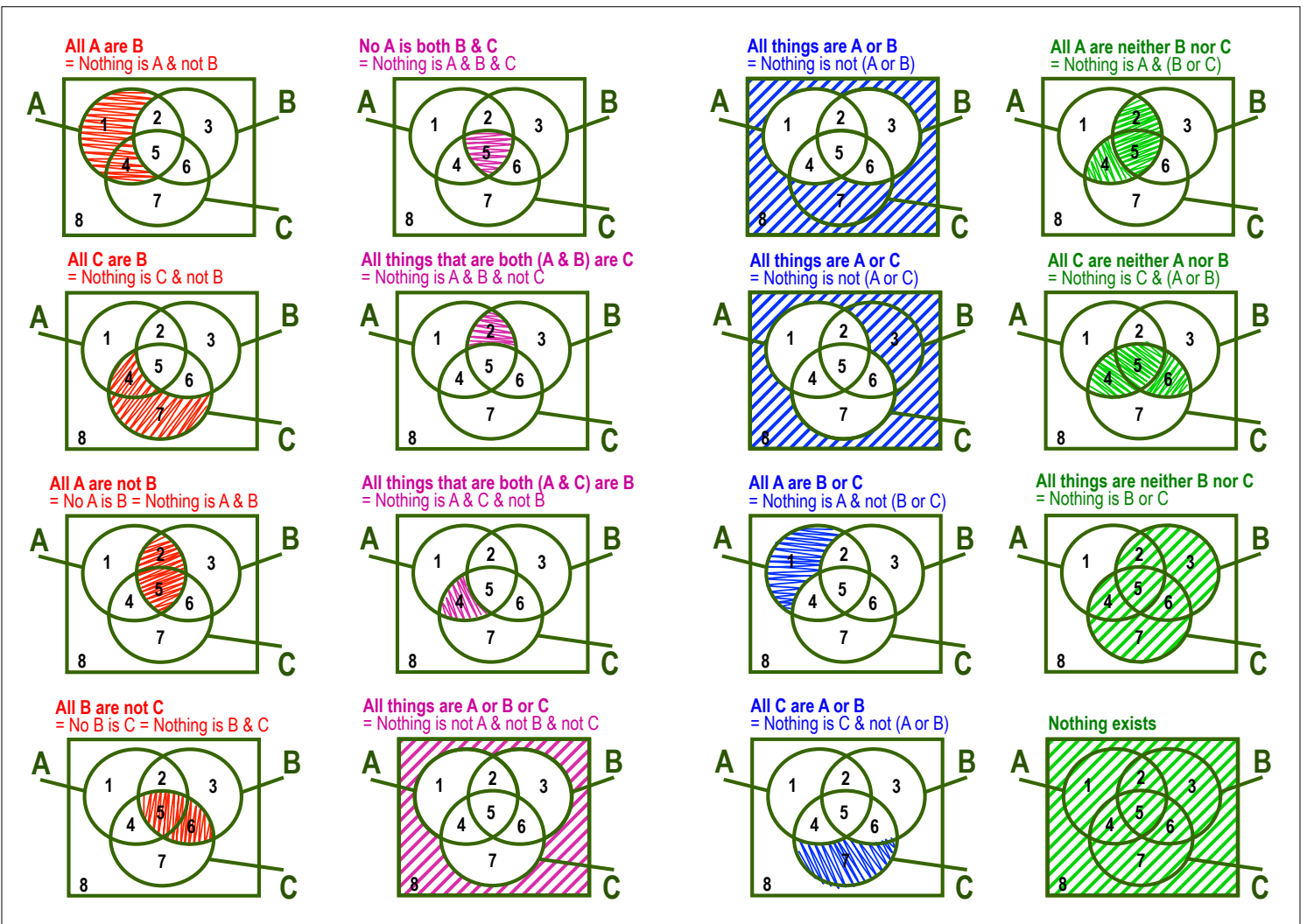
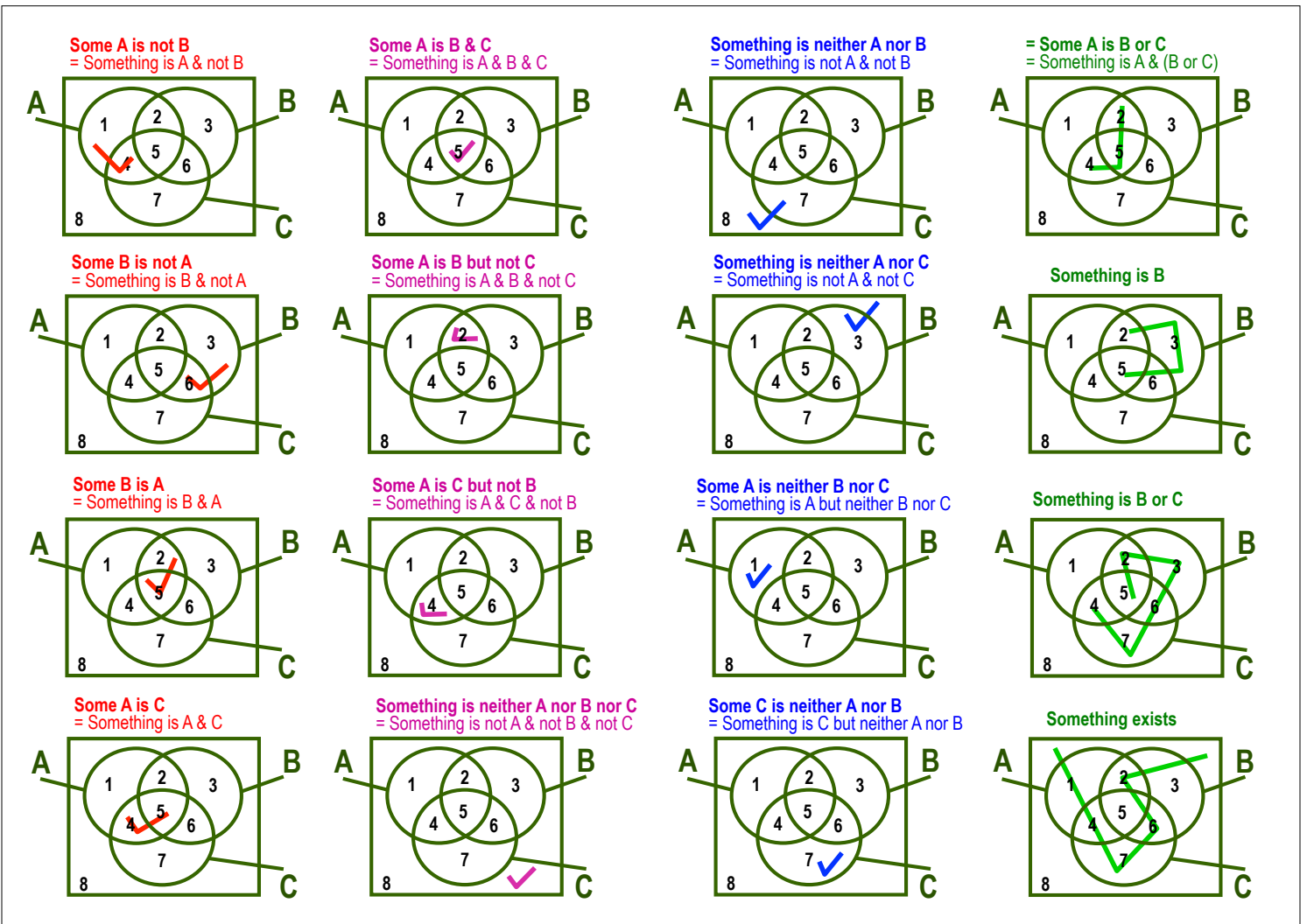
1, 2, 3, 5, 6, 8

$X \& (\text{not } Z \text{ or } Y)$

1, 2, 5

Tips

- ★ If different sets of regions are connected by “**or**”, then take the **union** of the sets (i.e., **all** the regions involved)
- ★ If different sets of regions are connected by “**and**”, then take the **intersection** (i.e., the **common** regions)



Use Venn Diagram to test whether the argument is valid

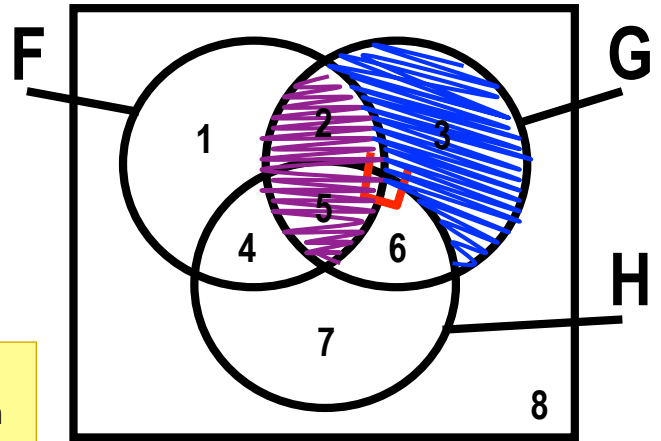
Example 11

P1. All G are F or H.

P2. Something is G.

P3. No F are G.

C. Some H are not-F.



- A tick is but a *crooked line*.
- When something exists in an area made up by more than two regions, we should use a *crooked line* to represent that.
- For instance, P2 amounts to saying that something exists in the area made up by regions 2, 3, 5, 6. So we should put a *crooked line* across all those regions.

- In order for the conclusion to be true ...
 - ... a *line* should appear *completely inside* the circle representing H *and completely outside* the circle representing F.
- That is indeed the case! Why?
- For regions 2, 3, 5 are all shaded, meaning that nothing exists in those regions. Given the replacement of the line (which means "something exists in 2 or 3 or 5 or 6") and the shading (which means "nothing exists in 2 or 3 or 5"), the result is "something is inside 6". So, we must read the position of the crooked line together with the shading as implying that something exists in region 6, which is a region *completely inside* the circle H *and completely outside* the circle F – meaning that *some H is not-F*.
- ∴ The argument is **VALID**.

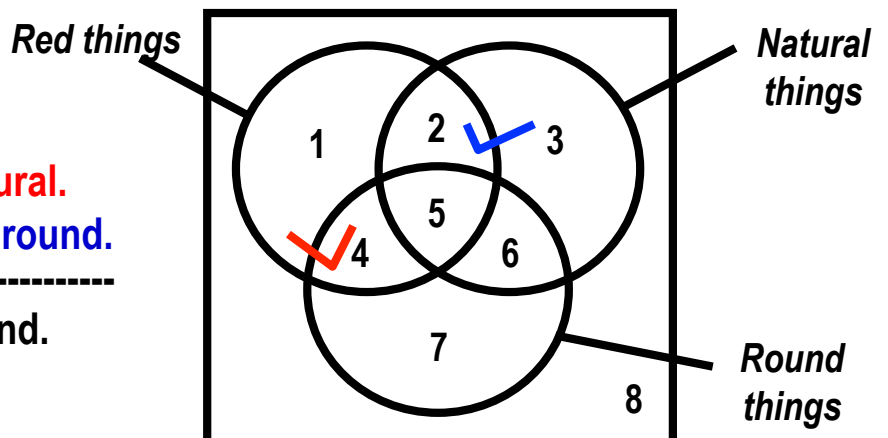
Use Venn Diagram to test whether the argument is valid

Example 12

P1. Not all red things are natural.

P2. Not all natural things are round.

C. Not all red things are round.



- In order for the conclusion to be true ...
 - ... a tick should appear *completely inside* the circle representing red things *and completely outside* the circle representing round things.
- That is **not** the case. No tick is such that it is completely inside the circle for red things and at the same time completely outside the circle for round things.
- ∴ The argument is **INVALID**.

Example 13

P1. Aristotle is not a musician but a logician.

P2. If one is a not philosopher, then one is a musician.

C. Aristotle is a philosopher.

(a) Represent all and only the premises in the diagram.

(b) The argument is .. **VALID** / INVALID ?

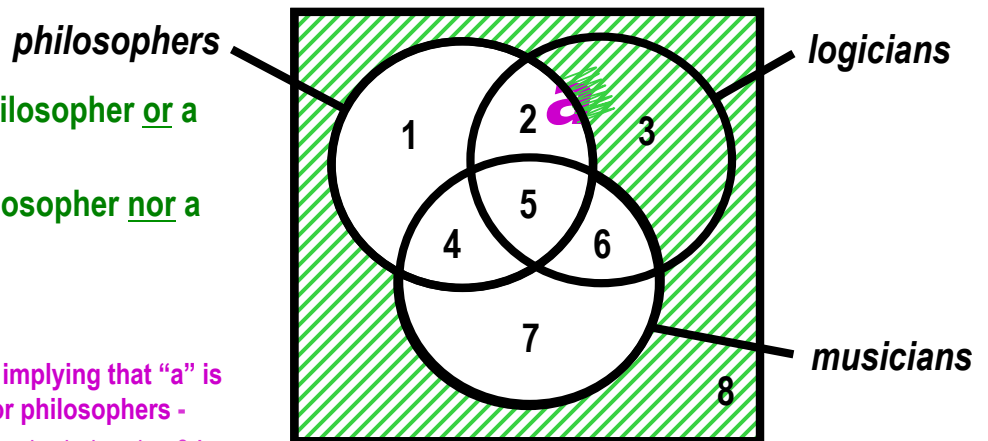
Hint:

P2 = Everyone is either a philosopher or a musician.

= No one is neither a philosopher nor a musician.

► **Shade regions 3 and 8.**

Note: We must read the diagram as implying that "a" is completely inside the circle for philosophers - because "a" cannot be inside the shaded region 3!



Use Venn Diagram to test whether the argument is valid

Example 14

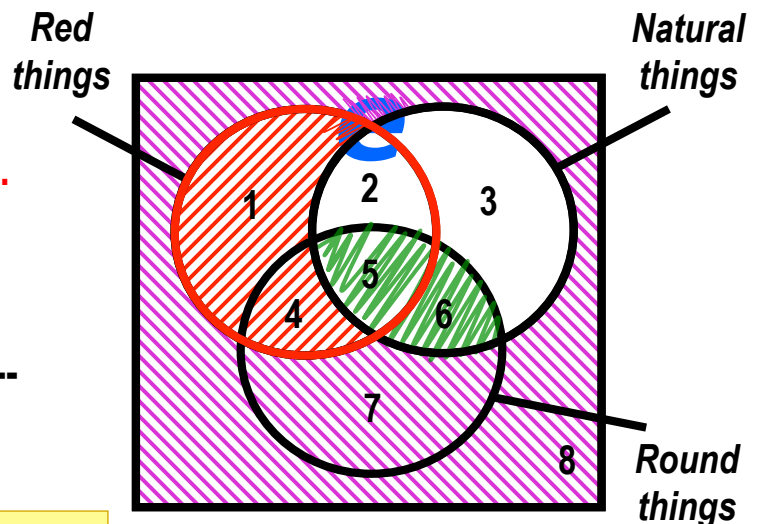
P1. If anything is red then it is natural.

P2. No round things are natural.

P3. Everything is natural or red.

P4. The Earth are not round.

C. The Earth are red.



Hint

"If anything is \square then it is \odot " = "All \square are \odot "

- In order for the conclusion to be true ...
... the "e" should appear **completely inside** the circle representing red things.
- That is not the case. For the position of "e" suggests that it can be in region 3 and not in region 2. There is no guarantee that it is completely inside the circle for read things.
- ∴ The argument is **INVALID**.

Use Venn Diagram to test whether the argument is valid

Example 15

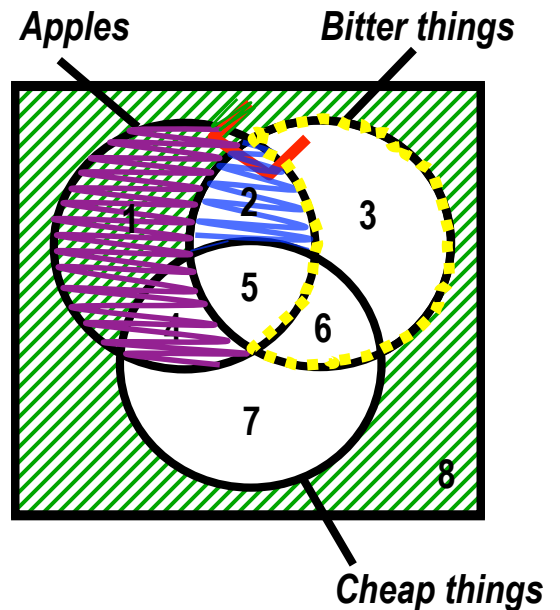
P1. Something is not cheap.

P2. If anything is an apple, then it is bitter.

P3. All bitter apples are cheap.

P4. Everything is bitter or cheap.

C. Some bitter thing is not an apple.



Hints

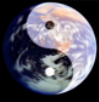
- 3 categories are mentioned. So, use 3 circles.
- You may use a crooked line instead if you cannot get a tick to cross all the required regions.
- “If anything is \square then it is \odot ” = “All \square are \odot ”

- In order for the conclusion to be true ...
... a line should appear **somewhere completely inside** the area representing things that are **bitter but not apples** (i.e., the area made up by regions 3 and 6).
(i.e., the area enclosed by dotted line)
- A line is indeed completely inside the area enclosed by the dotted line.
- ∴ The argument is **VALID**.

Summary

In this topic, we have learnt ...

- (1) Three kinds of statements that classify things into categories:
 - (a) **Universal** statements
 - (b) **Existential** statements
 - (c) **Singular** statements
- (2) Venn diagrams with one circle.
- (3) Venn diagrams with two circles.
- (4) Venn diagrams with three circles.
- (5) Using Venn diagrams to represent the above three kinds of statements.
- (6) Using Venn diagrams to test validity of arguments.



LOGIC

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Topic 5

END

