 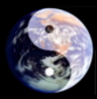    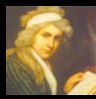   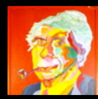 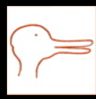

PHI2LOG LOGIC

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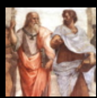
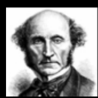
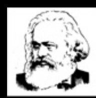
Topic 2

Four Basic Logical Connectives & Symbolization

Summary

Under Topic 2, we will learn to:

1. Recognize four basic **Logical Connectives** and their correspinding **Sentential Forms**:
   - **Conjunction** **(A & B)**
   - **Disjunction** **(A v B)**
   - **Conditional** **(A ⊃ B)**
   - **Negation** **(~A)**
2. Do **Symbolization**
   - Distinguish between **atomic statements** and **complex statements**
   - Use logical symbols to represent statements
   - Use logical symbols to represent arguments
3. Identify the **Main Connective** in a compound formula
4. Use **Formation Tree** to identify sub-formulas within longer formulas
5. Apply **Formation Rules** to construct **Well Formed Formulas** (WFFs)

Part 1. Four Basic Sentential Forms

3/09/2015

There are four basic **logical connectives** (also often called "logical **operators**"): "**and**, "**or**", "**if .... then ...**", "**not**". They are represented by the four different **logical symbols** "&", "v", "⊃", "~", respectively.

Logical Connectives	Symbols	Sentential Forms	Names (sentential forms)	Names (parts)
... **and** ...	**&**	**A & B**	**Conjunction**	A is the **first conjunct** B is the **second conjunct**
... **or** ... (inclusively)	**v**	**A v B**	**Disjunction**	A is the **first disjunct** B is the **second disjunct**
If ... **then** ...	⊃	**A ⊃ B**	**Conditional**	A is the **antecedent** B is the **consequent**
It is **not** true that ...	**~**	**~A**	**Negation**	

- The whole statement **"Ann is human and Bob is human"** has the logical form "A & B", where the component statement "Ann is human" occupies the A position, and "Bob is human" occupies the B position. The sentential form "A & B" is called the "**conjunction**". That which occupying position A is called the "first **conjunct**" and that which occupying position A is called the "second conjunct".
- **"Ann is human or Bob is human"** has the logical form "A v B". The sentential form "A v B" is called the "**disjunction**". That which occupying position A is called the "first **disjunct**" and that which occupying position A is called the "second disjunct".
- **"If Ann is human then Bob is human"** has the logical form "A ⊃ B". The sentential form "A ⊃ B" is called the "**conditional**". That which occupying position A is called the "**antecedent**" and that which occupying position A is called the "**consequent**".
- **"Ann is not human"** (or, equivalently, "it is not true that Ann is human") has the logical form "~A". The sentential form "~A" is called the "**negation**". "~A" is the negation of "A". The double negation "~~A" is the negation of "~A".

Part 2. Symbolization

3/09/2015

Use **small letters** to symbolise **atomic** statements (see slide 16 for more).	
Use **brackets** to disambiguate.	**( )**
... and ...	**&**
... or ... (inclusively)	**v**
If ... then ...	**⊃**
Not ...	**~**

Amu is pink if Amu is a pink emu.

= **If** Amu is a pink emu, **then** Amu is pink.

= **If** Amu is pink **and** Amu is an emu, **then** Amu is pink.

Symbolization

(p & e) ⊃ p

p = Amu is pink. e = Amu is an emu.

Memaw eats prawns if she eats tiger prawns.

= **If** memaw eats tiger prawns **then** memaw eats prawns.

Symbolization

t ⊃ p

p = Memaw eats prawns.

t = Memaw eats tiger prawns. [t ≠ Memaw eats tiger.]

Jay had chicken or fish or both.

Symbolization — " v " stands for inclusive "or".

c v f

c = Jay had chicken. f = Jay had fish.

Jay had chicken or fish.

Symbolization

c v f

NOTE: In this subject, as in many philosophy texts, the disjunction " **or** " (likewise, "**either ... or ...** " is used in the **inclusive sense**, where "A or B" means "A or B or both" – unless other indicated.

Jay had chicken or fish but not both.

Symbolization — " v̲ " stands for exclusive "or".

(c v f) & ~(c & f)

c v̲ f

Nobody is at home.

Symbolization

~h

h = Somebody is at home.

I think that nobody is at home

Symbolization

t

t = I think that nobody is at home.

Ben and John likes each other

Symbolization

b & j

b = Ben likes John.
j = John likes Ben.

1 and 1 together make 2

Symbolization

p

p = 1 and 1 together make 2.

Neither I nor you should believe him.

Symbolization

~(i v y)

~i & ~y

I = I should believe him.
y = You should believe him.

At least one of Amy, Bob and Cate is at home.

Symbolization

a v b v c

a = Amy is at home.
b = Bobby is at home.
c = Cat is at home.

It is untrue that Ghosts do not exist.

Symbolization

~~g

g = Ghosts exist.

Ann likes cats but Tom doesn't.

Symbolization

a & ~t

a = Ann likes cats. t = Tom likes cats.

I am going, if Mary is not going or John is not going.

= **If** Mary is **not** going **or** John is **not** going, **then** I am going.

Symbolization

(~m v ~j) ⊃ I

m = Mary is going. j = John is going.
i = I am going.

If you pay me then you will live, or you will die.

= **If** you pay me **then** you will live, **or else** you will die.

= **If** you pay me **then** you will live, **and if** you do **not** pay me **then** you will die.

= **If** you pay me **then** you will live, **and if** you do **not** pay me **then** you will **not live**.

Symbolization

(p ⊃ l) & (~p ⊃ ~l)

p = You pay me. **l** = You will live.

If you pay me, then you will live or you will die.

= **If** you pay me, **then** you will live **or** you will **not** live.

Symbolization

p ⊃ (l v ~l)

Position of the comma can be a matter of life and death!

A is a sufficient condition for B $=_{definition}$ **If A then B.**

Click to read more on the logic of "sufficient condition".

Your passing the exam is a **sufficient** condition for your passing the subject.

= **If** you pass the exam **then** you pass the subject.

Symbolization

e ⊃ s

e = You pass the exam. s = You pass the subject.

X's being female is a **sufficient** condition for X's being unable to become Pope.

= **If** X is female **then** X cannot become Pope.

Symbolization

f ⊃ ~p

f = X is female. p = X can become Pope.

Your passing the exam is **not** a **sufficient condition** for your passing the subject.

= It is **not** true that your passing the exam is a **sufficient condition** for your passing the subject.
= It is **not** true that **if** you pass the exam **then** you pass the subject.

Symbolization

~(e ⊃ s)

e = You pass the exam. s = You pass the subject.

A is a necessary condition for B $=_{df}$ **If not A then not B.**

Click to read more on the logic of "necessary condition".

X's having four sides is a **necessary** condition for X's being a square.

= **If** X does **not** have four sides, **then** X is **not** a square.

Symbolization

~f ⊃ ~s

f = X has four sides. s = X is a square.

X's being a female is a **necessary** condition for X's being a mother.

= **If** X is **not** female, **then** X is **not** a mother.

Symbolization

~f ⊃ ~m

f = X is female. m = X is a mother.

Your being good at sport is **not** a **necessary condition** for your being a good role model.

= It is **not** true that your being good at sport is a **necessary condition** for your being a good role model.
= It is **not** true that **if** you are **not** good at sport **then** you are **not** a good role model.

Symbolization

~(~s ⊃ ~r)

s = You are good at sport. r = You are a good role model.

A provided B $=_{df}$ **A if B.**
= If B then A.

He is happy **provided** that she is happy.
= He is happy **if** she is happy.
= **If** she is happy **then** he is happy.

Symbolization

s ⊃ h

s = She is happy. h = He is happy.

He is happy **provided** that she is unhappy.
= He is happy **if** she is unhappy.
= **If** she is unhappy **then** he is happy.

Symbolization

u ⊃ h

u = She is **unhappy**. h = He is happy.

You can enter and drink **provided** that you do **not** have a gun.
= You can enter and drink **if** you do **not** have a pass or a gun.
= **If** you do **not** have a gun, **then** you can enter and drink.
= **If** you do **not** have a gun, **then** you can enter **and** you can drink.

Symbolization

~g ⊃ (e & d)

g = You have a gun. e = You can enter. d = You can drink.

A unless B $=_{df}$ **A if not B.**
= If not B then A.

Click to read more on the logic of "unless".

I will be there **unless** I change my mind.
= I will be there **if** I do **not** change my mind.
= **If** I do **not** change my mind, **then** I will be there.

Symbolization

~c ⊃ b

c = I change my mind. b = I will be there.

Unless you believe, you will go to hell.
= **If** you do **not** believe, **then** you will go to hell.

Symbolization

~b ⊃ h

b = You believe. h = You will go to Hell.

You will **not** be happy **unless** you love yourself and others.
= You will **not** be happy **if** it is **not** the case that you love yourself and others.
= **If** it is **not** the case that you love yourself **and** you love others, **then** you will **not** be happy.

Symbolization

~(y & o) ⊃ ~h

y = You love yourself. a = You love others. h = You will be happy.

A only if B =$_{df}$ **Not A if not B.**

= **If not B then not A.**

logically equivalent to: **If A then B.**

Click to read more on the logic of "only if ".

You are in Beijing **only if** you are in China.

= You are **not** in Beijing **if** you are **not** in China.

= **If** you are **not** in China, **then** you are **not** in Beijing.

Symbolization

~c ⊃ ~b

Logically equivalent to: **b ⊃ c**

= **If** you are in Beijing, **then** you are in China.

c = You are in China. b = You are in Beijing

Rex is man **only if** Rex is **not** a dog.

= Rex is **not** a man **if** Rex is **not not** a dog.

= **If** Rex is **not not** a dog, **then** Rex is **not** a man.

= **If** Rex is a dog, **then** Rex is **not** a man.

Symbolization

d ⊃ ~m

Logically equivalent to: **m ⊃ ~d**

If Rex is a man, **then** Rex is **not** a dog.

d = Rex is a dog. m = Rex is a man.

A if and only if B = **A if B, and A only if B.**

= **If B then A, and A only if B.**

= **(B ⊃ A) & (A ⊃ B)**

= **A ≡ B**

- "A **iff** B" is the **short-hand** expression of "A if and only if B".
- The sentential form "A ≡ B" is called the "**biconditional**".

You can enter **iff** you do not have a gun.

Symbolization

e ≡ ~g

e = You can enter. g = You have a gun.

X is a father **iff** X is a male parent.

= X is a father **iff** X is male **and** X is a parent.

Symbolization

f ≡ (m & p)

f = X is a father. m = X is male. p = X is a parent.

Your having a ticket is a **necessary and sufficient condition** for your being able to enter.

Symbolization

(~t ⊃ ~e) & (t ⊃ e)

= **(e ⊃ t) & (t ⊃ e)**

= **e ≡ t**

= You can enter **if and only if** you have a ticket.

t = You have a ticket. e = You can enter.

Recall some well-known VALID argument forms (from Topic 1)

3/09/2015

1. Modus Ponens (MP)

```
A ⊃ B
A
-----------
B
```

2. Modus Tollens (MT)

```
A ⊃ B
~B
-----------
~A
```

3. Hypothetical Syllogism (HS)

```
A ⊃ B
B ⊃ C
-----------
A ⊃ C
```

4. Disjunctive Syllogism (DS)

```
A v B
~A
----------
B
```

```
A v B
~B
----------
A
```

5. Constructive Dilemma (CD)

```
A v B
A ⊃ C
B ⊃ D
-----------
C v D
```

6. Conjunction (Conj.)

```
A
B
----------
A & B
```

```
A
B
----------
B & A
```

7. Simplification (Simp.)

```
A & B
----------
A
```

```
A & B
----------
B
```

8. Addition (Add.)

```
A
---------
A v B
```

```
A
---------
B v A
```

9. De Morgan's Law (DM)

~(A & B) is logically equivalent to **~A v ~B**

```
~(A & B)
---------------
~A v ~B
```

```
~A v ~B
---------------
~(A & B)
```

~(A v B) is logically equivalent to **~A & ~B**

```
~(A v B)
---------------
~A & ~B
```

```
~A &~B
---------------
~(A v B)
```


We can also use shapes (instead of BIG letters) to represent the place-holders!

3/09/2015

1. Modus Ponens (MP)

```
○ ⊃ ■
○
------------
■
```

2. Modus Tollens (MT)

```
○ ⊃ ■
~■
------------
~○
```

3. Hypothetical Syllogism (HS)

```
○ ⊃ ■
■ ⊃ ★
------------
○ ⊃ ★
```

4. Disjunctive Syllogism (DS)

```
○ v ■
~○
-----------
■
```

```
○ v ■
~■
-----------
○
```

5. Constructive Dilemma (CD)

```
○ v ■
○ ⊃ ★
■ ⊃ ✚
------------
★ v ✚
```

6. Conjunction (Conj.)

```
○
■
-----------
○ & ■
```

```
○
■
-----------
■ & ○
```

7. Simplification (Simp.)

```
○ & ■
-----------
○
```

```
○ & ■
-----------
■
```

8. Addition (Add.)

```
○
----------
○ v ■
```

```
○
----------
■ v ○
```

9. De Morgan's Law (DM)

~(○ & ■) is logically equivalent to **~○ v ~■**

```
~(○ & ■)
----------------
~○ v ~■
```

```
~○ v ~■
----------------
~(○ & ■)
```

~(○ v ■) is logically equivalent to **~○ & ~■**

```
~(○ v ■)
----------------
~○ & ~■
```

```
~○ & ~■
----------------
~(○ v ■)
```

Part 3. Main Connective

Consider the following different formulas.

1. p ⊃ q
2. ~p ⊃ ~q
3. (p v q) ⊃ (r & s)
4. ~(p v q) ⊃ ((r & s) ⊃ t)

Question: What do the formulas have in common?

Answer: They all have the sentential form of the conditional: "**A** ⊃ **B** ".

The big letters "**A**" and "**B**" are just **place holders**.

For 1,	A = p	B = q
For 2,	A = ~p	B = ~q
For 3,	A = (p v q)	B = (r & s)
For 4,	A = ~(p v q)	B = ((r & s) ⊃ t)

The **main connective** in a formulas is the connective with the **largest scope** indicating the sentential form of the **whole** formula (i.e., the formula's **main sentential form**).

Highlights

Four basic **Logical Connectives**	Connective Symbols	Four basic **Sentential Forms**	Logical Symbolizations	Names (sentential forms)	Names (parts)	Alternative Symbols
It is not true …	~	It is not true that A	~A	Negation		¬
… and …	&	A and B	A & B	Conjunction	A = 1st Conjunct B = 2nd Conjunct	• ∧
… or … (or both)	v	A or B (or both)	A v B	Disjunction (inclusive)	A = 1st Disjunct B = 2nd Disjunct	
If … then …	⊃	If A then B	A ⊃ B	Conditional	A = Antecedent B = Consequent	→

Atomic statements are those that cannot be analysed as containing any logical connective.

Compound statements are those that can be analysed as containing at least one logical connective and at least one atomic statement.

Atomic formulas are the small letters (*p*, *q*, *r*, …etc.) used to represent atomic statements. (PHI2LOG convention)

Compound formulas are formulas that contain at least one logical connective symbol and at least one atomic formula.

Sub-formulas (i.e., *proper* sub-formulas) are the shorter formulas (atomic or compound) within a longer formula.

Brackets are used to as punctuation marks for disambiguating formula structure

Main connective in a compound formula is the connective with the largest scope, indicating the sentential form of the whole formula.

When we symbolize compound statements, we (usually) should display as much logical structure as possible – i.e., we should display as many logical connectives as possible whenever they occur. There are exceptions, however (see Slide 13).

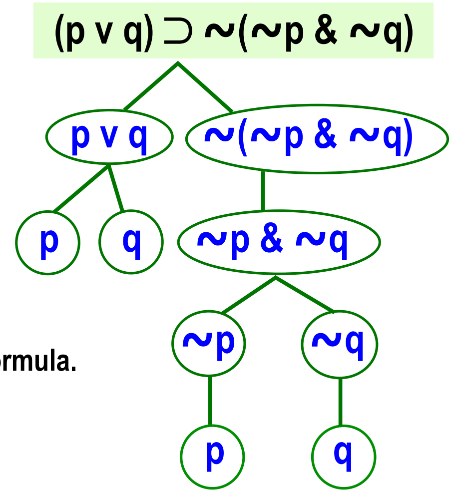
Part 4. Formation Tree (aka. Decomposition Tree)

This is a useful tool for laying out all the sub-formulas within any given compound formula.

Illustration:

Given any compound formula:

1) Identify the main connective in the formula, and
   * If it is a two-place connective, draw two branches pointing down.
   * If it is a one-place connective, draw one branch pointing down.
2) Put down the immediate sub-formulas joined by the connective.
3) For each sub-formulas identified, repeat the above two steps until the tree ends with only atomic formulas.



- **At each junction point or end point of the formation tree is a sub-formula.**
- **Some sub-formulas may have repeated occurrences.**

In the example:

- The sub-formulas "p " and "q " both have repeated occurrences.
- There are 7 sub-formulas in total within the original given formula (not counting repeated occurrences).
- 5 of them are compound sub-formulas.
- 2 of them are atomic sub-formulas.

For more examples, see Reading 2.1.

Amy will make sushi and Bobby will tidy up the house if Cate is visiting.

a = Amy will make sushi
b = Bobby will tidy up the house
c = Cate is visiting

The statement is *structurally ambiguous* (i.e., having more than one meaning due to its structure).

Symbolization (Interpretation #1)

Amy will make sushi, and if Cate is visiting then Bobby will tidy up the house.

a & (c ⊃ b)

The formula has the sentential form of the **conjunction**. Its main connective of the formula is " **&** ".

Symbolization (Interpretation #2)

If Cate is visiting, then Amy will make sushi and Bobby will tidy up the house.

c ⊃ (a & b)

The forumla has sentential form of the **conditional**. Its main connective of the formula is " ⊃ ".

If gods exist then demons exist.
Demons do not exist.

Gods do not exist.

Good/Useful Symbolization

g = Gods exist. **d** = Demons exist.

g ⊃ d
~d

~g [valid move - by Modus Tollens]

Bad/Useless Symbolization

p = If gods exist then demons exist.
q = Demons do not exist.
r = Gods do not exist.

p
q
----
r

[This symbolization does not help show whether the argument is valid.]

If gods exist or demons exist, then virtue exists or evil exists.
It is not true that virtue exists or evil exists.

Gods do not exist.

Useful/Good Symbolization

g = Gods exist. **d** = Demons exist. **u** = Virtue exists. **e** = Evil exists.

(**g** v **d**) ⊃ (**u** v **e**)
~(**u** v **e**)

~**g**

Also Acceptable Symbolization (for determining the argument's validity)

g = Gods exist. **d** = Demons exist. **p** = Virtue or evil exists.

1. (**g** v **d**) ⊃ **p**
2. ~**p**
3. ~(**g** v **d**) [valid move - from 1 and 2, by Modus Tollens]
4. ~**g** & ~**d** [valid move - from 3, by De Morgan's Law]

----

5. ~**g** [valid move - from 4, by Simplification]

Part 5. Well Formed Formula (WFF)

Formation Rules (for Propositional Logic / Sentential Logic)

R1. All sentence letters are WFFs.

R2. If A is a WFF, then ~A is a WFF.

R3. If A and B are WFFs, then (A & B), (A v B), (A ⊃ B), (A ≡ B) are also WFFs.

R4. Nothing else is a WFF (except for cases covered by the convention below)**.**

Convention (R5): For a WFF containing some set(s) of brackets, if dropping a set of brackets does not cause any ambiguity or change in meaning, then the formula resulting from dropping the set of brackets is a ***simplified version*** of the original WFF. But the dropped brackets will have to be restored before R2 or R3 can be applied to the WFF.

Atomic formulas "**p**" and "**q**" are WFFs by applying **R1**.
Compound formula "**~p**" is a WFF by applying **R2** to "p".
"**(p & q)**" is a WFF by applying **R3(&)** to "p" and "q". • Dropping the brackets in "(p & q)" will result in "**p & q**", which is the **simplified version** of the original WFF. • The bracket will have to be **restored** first if R3 is to be applied to the WFF. Example: By applying R3(v) to the restored "(p & q)" and to "~p", we get a new WFF "((p & q) v ~p)".
Dropping the *outer* brackets in "**((p & q) v ~p)**" will result in "**(p & q) v ~p**", which is the simplified version of the original WFF. Droppoing the *inner* brackets will result in the ambiguous "**(p & q v ~p)**", which is not a WFF at all.
"**~(p & q)**" is a WFF. Dropping the brackets will result in "**~p & q**", which has a different meaning and so is not the simplified version of the original WFF "~(p & q)". Instead, "~p & q" is the simplified version of the WFF "**(~p & q)**".

Highlights

	Logical Symbolization	*Equivalent Form*	
B provided A (= A if B)	A ⊃ B		
A is a sufficient for B	A ⊃ B		
B is a necessary for A	~B ⊃ ~A	A ⊃ B	
A only if B (= Not A if not B)	~B ⊃ ~A	A ⊃ B	
A provided B (= A if B)	B ⊃ A		
B is a sufficient for A	B ⊃ A		
A is a necessary for B	~A ⊃ ~B	B ⊃ A	
B only if A (= Not A if not B)	~A ⊃ ~B	B ⊃ A	
A is a necessary and sufficient for B	(~A ⊃ ~B) & (A ⊃ B)	(B ⊃ A) & (A ⊃ B)	A ≡ B
A if and only if B (shorthand: A iff B)	(B ⊃ A) & (~B ⊃ ~A)	(B ⊃ A) & (A ⊃ B)	A ≡ B
A unless B (= A if not B)	~B ⊃ A		
Neither A nor B (= Not either)	~(A v B)	~A & ~B	

Summary

Under Topic 2, we have learnt to:

1. Recognize four basic **Logical Connectives** and their correspinding **Sentential Forms**:
   - **Conjunction** **(A & B)**
   - **Disjunction** **(A v B)**
   - **Conditional** **(A ⊃ B)**
   - **Negation** **(~A)**
2. Do **Symbolization**
   - Distinguish between **atomic statements** and **complex statements**
   - Use logical symbols to represent statements
   - Use logical symbols to represent arguments
3. Identify the **Main Connective** in a compound formula
4. Use **Formation Tree** to identify sub-formulas within longer formulas
5. Apply **Formation Rules** to construct **Well Formed Formulas** (WFFs)